J and CTOD Estimation for Homogeneous and Bi-Material Fracture Toughness Testing Specimens

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This paper proposes J and CTOD estimation schemes applied to fracture toughness testing, covering typical homogeneous and bi-material specimens. Recommendations are based on the plastic limit analysis (either slip line field or finite element limit analyses), assuming the rigid plastic material behavior. The main outcome of the present study is that the J and CTOD estimation schemes (both codified and non-codified), recommended for homogeneous specimens, can be equally used for bi-material specimens with interface cracks. The effect of yield strength mismatch in bi-material specimens on the the J-integral and CTOD is discussed.

Key Words : Bi-material Specimen, Crack Tip Opening Displacement, Interface Crack, J-integral, Limit Load, Strength Mismatch, Fracture Toughness Testing

1. Introduction

Application of the fracture mechanics to assessments of structural integrity requires the evaluation of both "applied" and "resistances" side. For the applied side, crack driving force parameters such as the stress intensity factor, K, the J-integral, and the crack-tip opening displacement (CTOD), δ, should be estimated in terms of a load, for a given defective structure. On the other hand, the resistance to cracking of a material (fracture toughness) should be found, which then can be compared with the applied side to assess the significance of the flaw in the structure.

To evaluate the toughness of a material, specific toughness testing procedures should be followed according to the standardized methods. Up to date, all of fracture toughness testing standards (ASTM Standards; British Standards, BS 5447;

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ISO/CD 12135), are applicable to the homogeneous specimens. In practice, however, measuring the toughness properties of bimaterial specimens with interface cracks is of concern, like the heat affected zone toughness of bi-metallic joints or of weldments. At the present, any codified testing procedure for such nonhomogeneous specimens is not available except several standards in a draft form (ASTM E1290; British Standards, BS 7448; ISO/CD 15653). Hence, the present paper proposes some recommendations on J and CTOD estimations for a general fracture toughness testing of homogeneous specimens and bi-material joints with interfacial cracks.

2. Background of J and CTOD Estimation

2.1 Codified exercise

It is a typical practice in toughness testing to estimate the J-integral as

$$J = J_e + J_p$$
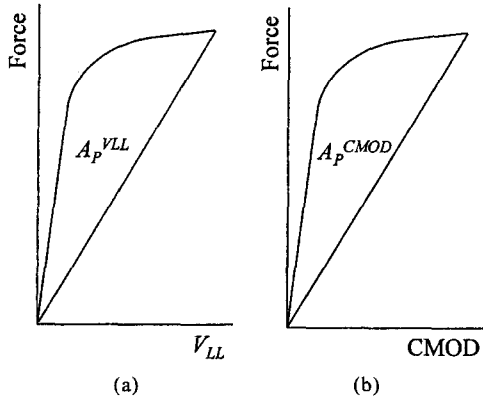$$= \frac{(1-\nu^2)K^2}{E} + \eta_p^{VLL} \cdot \frac{A_p^{VLL}}{B(W-a)} \qquad (1)$$

Fig. 1 Schematic drawings for the plastic area of (a) load-load line displacement and (b) load-CMOD curves

where B, W, a denote the thickness, width and crack depth of specimen, respectively. J_e, the elastic part of J, is evaluated from the stress intensity factor (K), Young's modulus (E), and Poisson's ratio (ν). The plastic part J_p is evaluated from the plastic η factor (η_p^{VLL}), and the plastic portion of the load-load line displacement curve (A_p^{VLL}) of Fig. 1(a). Here the force in Fig. 1 means the load-line force of the specimen and VLL the load-line displacement. The factor (η_p^{VLL}) depends strongly on the specimen type and the crack depth, but its dependence on the material (strain hardening) is minimal (Kim and Schwalbe, 2001a). For instance, for deeply cracked single-edge-notched bend [SE(B)] specimens with the crack depth of a/W=0.45~0.75[see Fig. 2(a)], η_p^{VLL}=2 is recommended in the current standards (ASTM standards; BS 5447; ISO/CD 12135).

According to the load-separation principles (Sharobeam and Landes, 1991; Sharobeam and Landes, 1993), the dependence of η_p^{VLL} on the specimen type and the crack depth can be found in limit load solutions:

$$\eta_p^{VLL} = -\frac{(W-a)}{P_L} \cdot \frac{\partial P_L}{\partial a} \qquad (2)$$

where P_L denotes the plastic limit load, and a function of a/W. Therefore, once the limit load of the particular specimen of interest is found, then η_p^{VLL} can be evaluated according to Eq. (2).
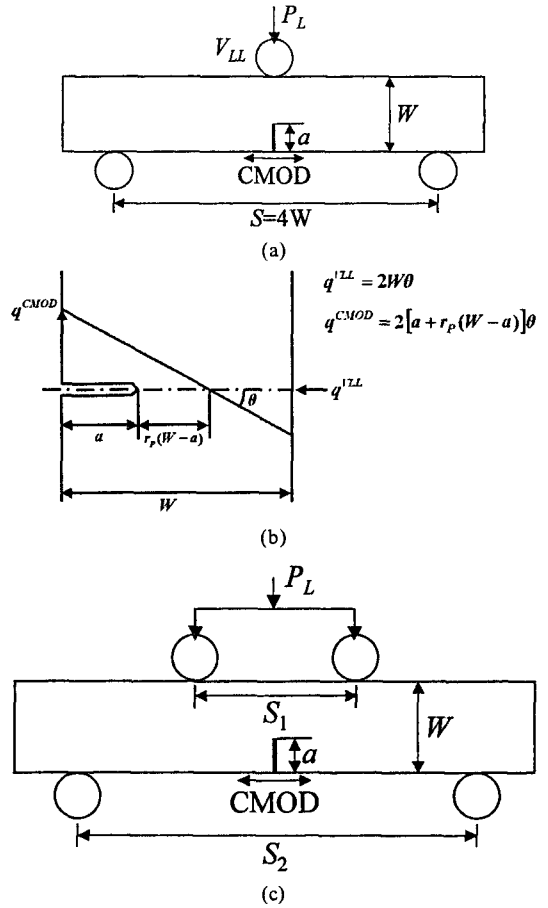

Fig. 2 Schematic drawings for (a) single edge cracked specimen in three-point bend [SE(B)], (b) relationship between the load line displacement and the CMOD, and (c) in four-point bend [SE(PB)]

The codified standards also provide the CTOD estimation scheme from the plastic component of the crack mouth opening displacement (CMOD), δ_{mp}, simply using the rotation factor, r_p :

$$\delta = \delta_e + \delta_p$$
$$= \frac{(1-\nu^2)K^2}{2E\sigma_Y} + \frac{r_p(W-a) \cdot \delta_{mp}}{r_p(W-a)+a+z} \qquad (3)$$

where σ_Y is the yield strength and z the height of knife edge attached to the specimen. For deeply cracked SE(B) specimens, the standards recommend either r_p=0.4 (BS 5447) or 0.44 (ASTM Standards). Notice that the different standards above provide the different values of r_p; these two values may lead to about 8% difference in δ.

However, it must be noted that the r_p value depends strongly on the strain hardening of material. Thus such difference is regarded as insignificant in practical fracture toughness testing.

2.2 Non-codified exercise

Recently, it has been argued that the use of the plastic portion of the load-crack mouth opening displacement, A_p^{CMOD} [see Fig. 1(b)], could lead to a better estimation of the J-integral (Kirk and Dodds, 1993; Kirk and Wang, 1995; Wang and Gordon, 1992) as in Eq. (4).

$$J=J_e+J_p$$
$$=\frac{(1-\nu^2)K^2}{E}+\eta_p^{CMOD}\cdot\frac{A_p^{CMOD}}{B(W-a)} \qquad (4)$$

Assuming that the material follows rigid plastic behavior, it is easily shown that η_p^{CMOD} is related to η_p^{VLL} by the rotation factor r_p and other related specimen dimensions, and thus explicit relationships between η_p^{CMOD} and η_p^{VLL} depend on the specimen geometry and loading types. For instance, consider SE(B) specimens, as depicted in Fig. 2(a). A rigid body rotation deformation [Fig. 2(b)] provides the relationship between the load line displacement, q^{VLL}, and the CMOD, q^{CMOD}:

$$q^{CMOD}=\left[\frac{a}{W}+r_p\left(1-\frac{a}{W}\right)\right]q^{VLL} \qquad (5)$$

For a rigid-plastic, non-hardening material, the J-integral can be expressed as

$$J=\eta_p^{VLL}\frac{P_L\ q^{VLL}}{B(W-a)}=\eta_p^{CMOD}\frac{P_L\ q^{CMOD}}{B(W-a)} \qquad (6)$$

Combining Eqs. (5) and (6) gives a relationship between η_p^{CMOD} and η_p^{VLL} for SE(B) specimens as

$$\eta_p^{CMOD}=\frac{\eta_p^{VLL}}{\left[\frac{a}{W}+r_p\left(1-\frac{a}{W}\right)\right]} \qquad (7)$$

Consequently, it has been proposed that the CTOD, δ, can be estimated directly from the plastic component J_p of the estimated J-integral [see Eq. (4)]

$$\delta=\delta_e+\delta_p=\frac{(1-\nu^2)K^2}{2E\sigma_Y}+\frac{J_p}{m\sigma_f} \qquad (8)$$

where σ_f denotes the flow strength, typically defined as the average of the yield and the tensile

strengths, $\sigma_f=(\sigma_y+\sigma_u)/2$, and m is the constraint factor, relating δ_p and J_p:

$$m=J_p/(\sigma_f\delta_p) \qquad (9)$$

2.3 Present analysis

The present work proposes formulae for J and CTOD estimations, such as η_p^{VLL}, η_p^{CMOD} and m factors for toughness testing specimens. Those formulae have been developed from the plastic limit analysis such as the slip line field (SLF) or detailed finite element (FE) limit analyses, based on rigid-perfectly plastic material behavior. Firstly, for a given specimen geometry, the limit load solutions were obtained from SLF and FE limit analyses, and then η_p^{VLL} can be found by Eq. (2). Moreover, such limit analyses also calculate the plastic rotation factor, r_p, and thus η_p^{CMOD}, is estimated by Eq. (7) for SE(B) specimens. The rigid-plastic model can also provide an explicit relationship between J_p and δ_p, which gives the m-factor by using Eq. (9). Thus in principle, if the solutions for P_L and r_p are given, then the solutions for η_p^{VLL}, η_p^{CMOD} and m can be found. In the following, these solutions are given for typical homogeneous fracture toughness testing specimens.

3. Results for Homogeneous Specimens

Before presenting results of various mismatched specimens, it is instructive to compare the present results with those in literature and to validate the approach taken in the present work. Consider a three point bend specimen [SE(B)], Fig. 2(a), of which the J and CTOD estimation schemes are widely available. For this specimen, the SLF solutions on P_L and r_p have been given by Wu *et al.* (1988) and Wu *et al.* (1990). Figure 3 compares the values of η_p^{VLL}[Eq. (2)], η_p^{CMOD}[Eq. (7)] and m [Eq. (9)], determined from the present analysis, with those available in literature (see the legend in the figure). It shows that the present analysis provides a quite good approximation for $a/W>0.2$. Note that for SE(B) specimens the value of $a/W=0.2$ is characterized as the boundary between the deep and shallow
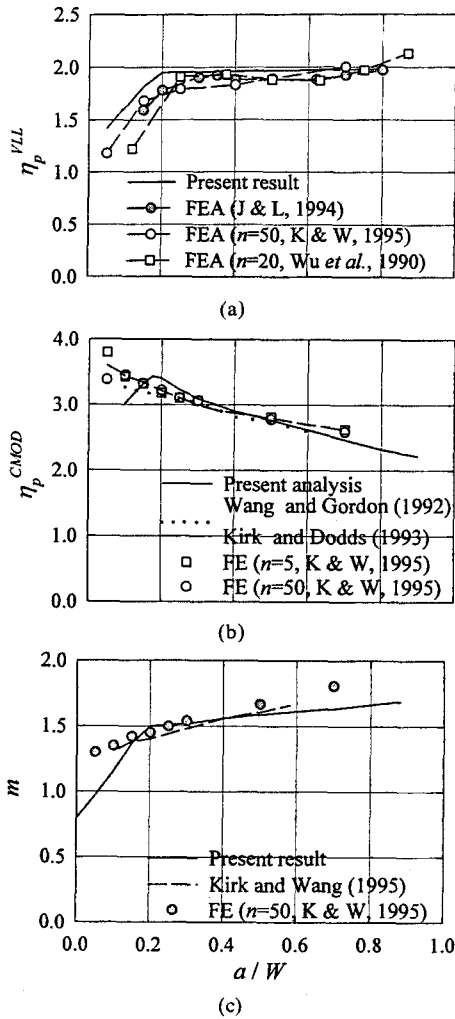
Fig. 3 Results for the homogeneous SE(B) specimens: (a) η_p^{VLL} factor, (b) η_p^{CMOD} factor, and (c) m factor

cracks (Wu *et al.*, 1990). For shallow cracks, $a/W < 0.2$, the present analysis does not provide the results consistent with those in literature. The reason is as follows. The assumption of a rigid-plastic material, being a basis of the SLF analysis, is appropriate only when the plastic strain dominates the elastic strain. For the deeper cracks, a small deformation would be sufficient for dominant plastic strain, but for the shallower cracks, much larger deformation is necesary due to the plasticity spreading to the back surface. Note that all FE results shown in Fig. 3 were obtained at a certain level of deformation which

may not be sufficiently large (Joyce and Link, 1994; Kirk and Dodds, 1993; Kirk and Wang, 1995; Wang and Gordon, 1992; Wu *et al.*, 1990). Thus, the results in Fig. 3 suggest that the results for η_p^{VLL} be in general reliable for all a/W. The results of η_p^{CMOD} are acceptable at least for deep cracks, $0.2 < a/W < 0.7$. On the other hand, those for m are good only for $0.2 < a/W < 0.5$. For $0.5 < a/W < 0.7$, the proposed values of m differ from the existing solution by up to 10%. Typically it is known that the value of m depends not only on the strain hardening but also on the load magnitude, and thus caution should be given to estimate the m value from the FEA results. We believe that such difference results from the fact that Kirk and Wang (1995) extracted their values of m for $0.5 < a/W < 0.7$ at too high loads, as pointed out recently by Kim and Schwalbe (2001a), who found that the solutions by Kirk and Wang are not so accurate for $0.5 < a/W < 0.7$, particularly for m. Considering sensitivity of m in typical testing, even 10% difference should be regarded as "not significant". Therefore, it is concluded that the proposed solutions are reliable at least for deep cracks. Moreover, noting that in the current testing standards (ASTM Standards; British Standards, BS 5447; ISO/CD 12135), the permissible range for the crack length is $0.45 < a/W < 0.7$ for ensuring sufficiently high crack tip constraint, this paper concentrates on the solutions for various specimens with $0.45 < a/W < 0.7$. As noted, the present results for these range of the crack lengths should be reliable in practical toughness testing.

Figure 2(c) depicts a single edge cracked specimen in pure (four point) bend, SE(PB). Although it is not recommended in the testing standards, such geometry may have certain advantages for bi-material specimens, as discussed in the next section. Figure 4(a) compares the present FEA limit load solutions of the SE(PB) specimens with the regression curve to the results from the slip line field analysis, proposed by Wu *et al.* (1990):

$$\frac{P_L \cdot (S_1 - S_2)/2}{(\sigma_Y/\sqrt{3})(W-a)^2}$$

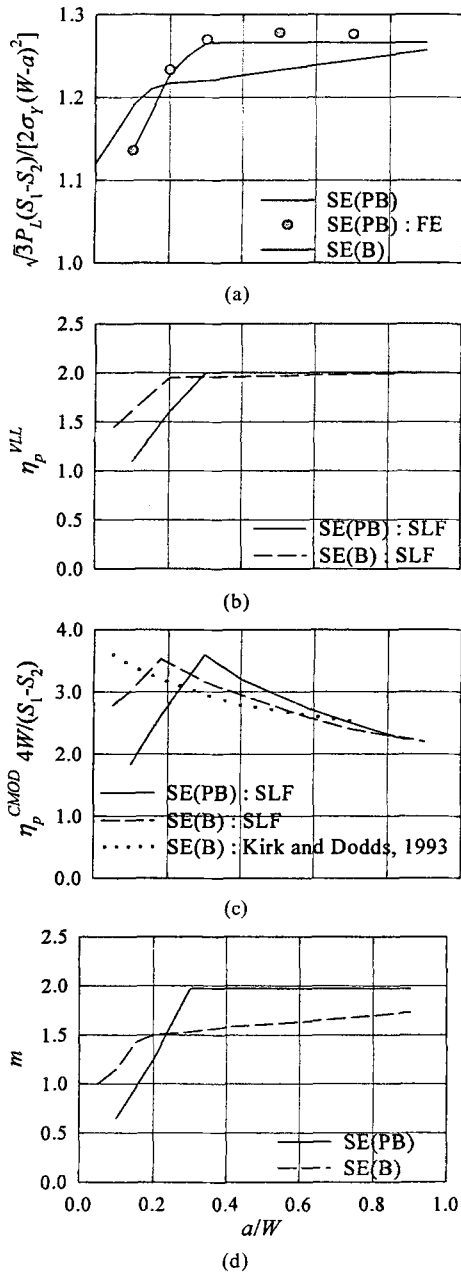which shows excellent agreements. They are also compared with the solutions for the SE(B) specimens. The resulting values for η_p^{VLL}, calculated from Eq. (2), are shown in Fig. 4(b). Noting that the limit loads do not depend on a/W for the deep crack range $(0.45 \le a/W \le 0.7)$, $\eta_p^{VLL}=2$ which has been already shown by Rice *et al.* (1973). The values of r_p for SE(PB) specimens have also been obtained based on the detailed FE limit analyses by Lee and Parks (1993). The values of η_p^{CMOD}, obtained using these r_p values, are shown in Fig. 4(c). For $0.45 \le a/W \le 0.7$, the results of SE(PB) can be approximated by the following linear equation:

$$\frac{\eta_p^{CMOD} \cdot 4W}{(S_1-S_2)} = 4.31 - 2.75\left(\frac{a}{W}\right) \qquad (11)$$

The resulting *m*-factor is shown in Fig. 4(d). For deeply cracked bend specimens, it is known that $m=2$ (Rice *et al.*, 1973), which can be clearly seen in Fig. 4(d).

4. Results for Interfacially Cracked Bi-material Specimens

In this section, the analysis is extended to bi-material specimens with interface cracks. The geometries of which are shown in Fig. 5: SE(PB), compact tension (C(T)), and middle cracked tension (M(T)) specimens. The SE(B) specimen is not discussed here. Due to the strength mismatch, asymmetric deformation pattern is expected for bi-material specimens because the lower strength material deforms larger than the higher strength one. The asymmetric deformation can give a problematic situation for the SE(B) specimen in that the roller may not remain in the cracked plane, and thus promote the mixed mode conditions. Thus, for testing bi-material specimens with interface cracks, other geometries such as the SE(PB) or C(T) specimens may be more appropriate. The M(T) specimen is also useful to investigate the constraint effect on fracture toughness, which is a low constraint geometry.

Bi-material specimens are characterized as mismatch in material properties of two constituents. The elastic property mismatch does

Fig. 4 Results for the homogeneous SE(PB) specimens and SE(B) specimens; (a) limit load PL, (b) η_p^{VLL} factor, (c) η_p^{CMOD} factor, and (d) m factor

$$=\begin{cases} 1+1.62\left(\dfrac{a}{W}\right)-2.49\left(\dfrac{a}{W}\right)^2 & 0<\dfrac{a}{W}<0.3 \\ 1.262 & 0.3\le\dfrac{a}{W}<1.0 \end{cases} \qquad (10)$$
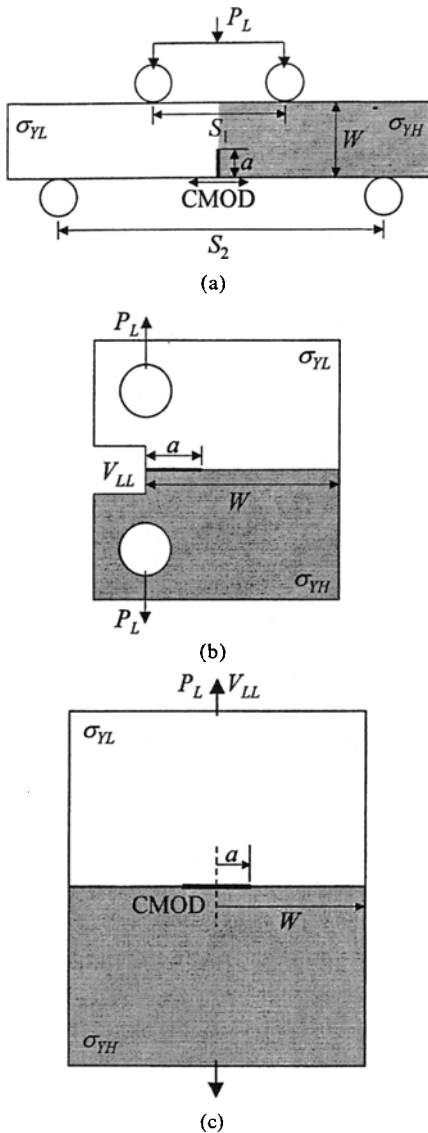
Fig. 5 Schematic drawing for bi-material specimens with interface cracks: (a) four-point bend [SE(PB)], (b) compact tension [C(T)], and (c) middle cracked tension [M(T)] specimens

affect the J and CTOD values only in contained yielding, and does not affect those in wide spread plasticity. Such elastic mismatch can be incorporated in elastic quantities (J_e and δ_e) of J and CTOD. Thus, of main concern is the plastic mismatch, particularly the mismatch in yield strengths, "strength mismatch". The strength mismatch can be quantified in terms of the factor,

M, defined by

$$M \equiv \frac{\sigma_{YH}}{\sigma_{YL}} \left(>1 \right) \tag{12}$$

where σ_{YH} and σ_{YL} denote the yield strength associated with higher and lower yield strength materials of a bi-material specimen, respectively. Note that $M=1$ means homogeneous specimens, and $M=\infty$ is as to elastic material bonded to elastic plastic material. A simple limit analysis can easily show that the limit loads for intermediate values of M must be bounded between those for $M=1$ and for $M=\infty$. The authors have also checked it using detailed FE limit analyses. It should be also noted that the plastic limit analysis for the bi-material specimens show that the plastic limit load is controlled by the lower strength material, and thus the limit load solutions in the subsequent figures are normalized with respect to the yield strength of the lower strength material, σ_{YL}.

Figure 6(a) compares the limit load solutions of the SE(PB) specimens for two extreme M values, $M=1$ and $M=\infty$, together with the results from detailed FE analyses for $M=\infty$. As mentioned above, results in Fig. 7(a) are normalized with respect to the yield strength of the lower strength material, σ_{YL}. Based on these FE results, the following approximation for the limit load, P_L, may be proposed for $0.45 \leq a/W \leq 0.7$:

$$\frac{P_L \cdot (S_1 - S_2)/2}{(\sigma_{YL}/\sqrt{3})(W-a)^2}$$

$$= \begin{cases} 1 + 1.731\left(\frac{a}{W}\right) - 2.164\left(\frac{a}{W}\right)^2 & 0 < \frac{a}{W} < 0.4 \\ 1.34 & 0.4 \leq \frac{a}{W} < 1.0 \end{cases} \tag{13}$$

Note that the limit load for $M=\infty$ does not depend on a/W for $0.45 \leq a/W \leq 0.7$, as for the homogeneous ($M=1$) specimens. The limit load for $M=\infty$ is at most 6% higher than that for corresponding homogeneous specimens for deep cracks. For the intermediate values of M, it has been shown that the limit loads for $0.45 \leq a/W \leq$ 0.7 do not depend on a/W (Kim and Schwalbe, 2001b), based on detailed FE limit analyses. This means that $\eta_p^{VLL} = 2$ can be used for SE(PB)
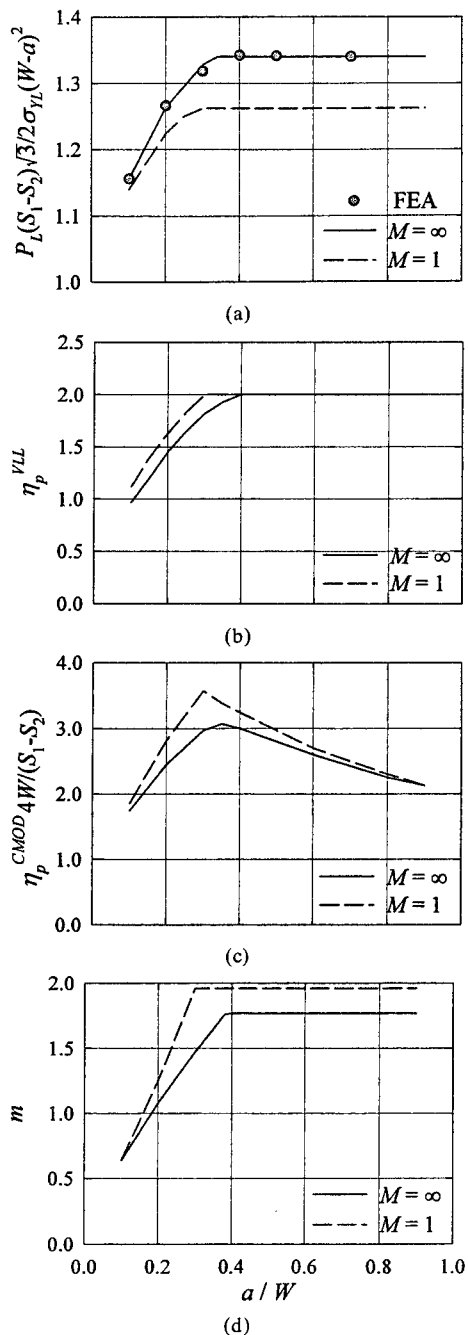
Fig. 6 Results for the 4PTB specimens: (a) limit load P_L, (b) η_p^{VLL} factor, (c) η_p^{CMOD} factor, and (d) m factor

bi-material specimens with $0.45 \leq a/W \leq 0.7$, regardless of M values, as shown in Fig. 6(b). The plastic rotation factor solutions for the SE(PB) bi-

material specimen with $M = \infty$ were found by Kim (1994) using detailed FE limit analyses. Based on these FE results of r_p, the values of η_p^{CMOD} for $M = \infty$ can be calculated, which are shown in Fig. 6(c), which can be approximated linearly for $0.45 \leq a/W \leq 0.7$ as

$$\frac{\eta_p^{CMOD} \cdot 4W}{(S_1 - S_2)} = 3.86 - 2.14\left(\frac{a}{W}\right) \tag{14}$$

The suggested approximation, Eq. (14), gives values less than 6% of those for homogeneous SE (PB) specimens, $M = 1$. In principle, for the intermediate values of M, similar procedures can be taken, as the plastic limit load and r_p solutions are available. Without detailed analysis, it can be easily shown that the resulting values of η_p^{CMOD} for the intermediate values of M will be within 6% of those for homogeneous SE(PB) specimens, $M = 1$. Such error is insignificant when considering many other factors involved in fracture toughness testing and thus the homogeneous testing procedure can be used to testing every bi-material specimen with an interface crack. The m-factor for $M = \infty$ can be calculated from Eq. (9) using the FEA results of J_p and r_p and the slip line field analysis and the resulting values are shown in Fig. 6(d). For $0.45 \leq a/W \leq 0.7$, $m = 1.8$ for $M = \infty$ which is about 10% lower than that for homogeneous SE(PB) specimens $(M = 1)$. Again the 10% difference in m (and thus the CTOD determination) should be regarded as insignificant in CTOD testing, and thus the present homogeneous testing procedure may be used for the bi-material specimens with interface cracks.

Figure 7(a) compares the limit load solutions of the C(T) specimens for three different values of M: $M = 1$, $M = 1.5$, and $M = 2$. Again, the results in Fig. 7(a) are normalized with respect to the yield strength of the lower strength material, σ_{YL}. The coincidence of the results of three different M values suggests that the plastic limit load solutions do not depend on the yield strength of the higher strength material, and the limit load for bi-material is the same as that for homogeneous C(T) specimen made of the lower strength material. For $0.45 \leq a/W \leq 0.7$, the following regression equation can be used for all M values:
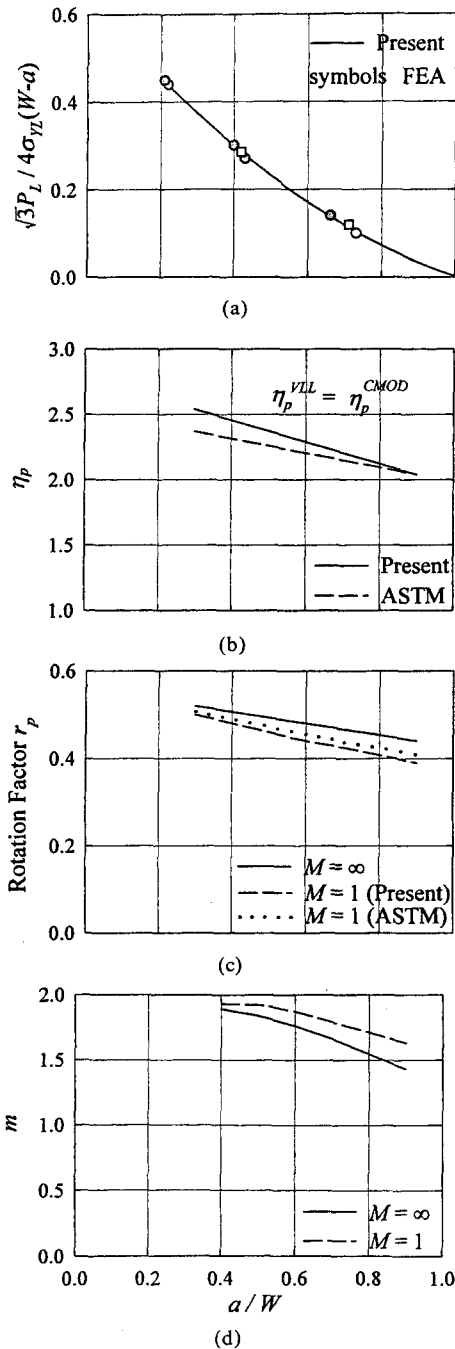
Fig. 7   Results for the CT specimens: (a) limit load P_L, (b) η_p^{VLL} and η_p^{CMOD} factor, (c) rotation factor r_p, and (d) m factor

$$\frac{\sqrt{3}P_L}{\sigma_{YL}(W-a)}=0.64-1.04\left(\frac{a}{W}\right)+0.4\left(\frac{a}{W}\right)^2 \quad (15)$$

The resulting η_p^{VLL} value for all M values are

shown in Fig. 7(b), which is linear with a/W between $0.45 \le a/W \le 0.7$:

$$\eta_p^{VLL}=2.78-0.81\left(\frac{a}{W}\right) \quad (16)$$

Note that in practice for the C(T) specimen the load line displacement is measured from the clip gauge attached to the stepped notch, and thus is close to CMOD. Therefore, for the C(T) specimen, the value of η_p^{VLL} is same as that of η_p^{CMOD}:

$$\eta_p^{VLL}=\eta_p^{CMOD}=2.78-0.81\left(\frac{a}{W}\right) \quad (17)$$

Figure 7(b) also includes the current recommendations in the standards:

$$\eta_p^{VLL}=2.0+0.522\left(1-\frac{a}{W}\right) \quad (18)$$

which is slightly (about 5%) lower than and thus is close to the present result. Figure 7(c) shows the results of rotation factor, r_p for two extreme values of M: $M=1$ and $M=\infty$, which can be approximated for $0.45 \le a/W \le 0.7$ as

$$r_p(M=\infty)=0.553-0.125\left(\frac{a}{W}\right) \quad (19)$$

$$r_p(M=1)=0.60-0.36\left(\frac{a}{W}\right)+0.13\left(\frac{a}{W}\right)^2 \quad (20)$$

These two approximations are also compared with the current ASTM recommendation in Fig. 7 (c). All the values shown in Fig. 7(c) are only within 8%. As said, the value of r_p strongly depends on the strain hardening and thus such difference is insignificant in practice. Figure 7(d) shows that the m values for the two extreme values of $M=1$ and $M=\infty$ differ from about 6%, which again can be regarded as close values from a toughness testing view point. The following approximations can be used for $0.45 \le a/W \le 0.7$:

$$m(M=\infty)=2.0+0.05\left(\frac{a}{W}\right)-0.77\left(\frac{a}{W}\right)^2 \quad (21)$$

$$m(M=1)=1.71+1.13(a/W)-1.36(a/W)^2 \quad (22)$$

Figure 8 shows the effect of the strength mismatch, M on the limit load, P_L for the M(T) specimens. Increasing M increases the limit load up to 30%. However, for a given M value, P_L does not change with a/W. Moreover, the rotation factor is not affected by M. Therefore, for the M (T) specimens, the scheme for the homogeneous
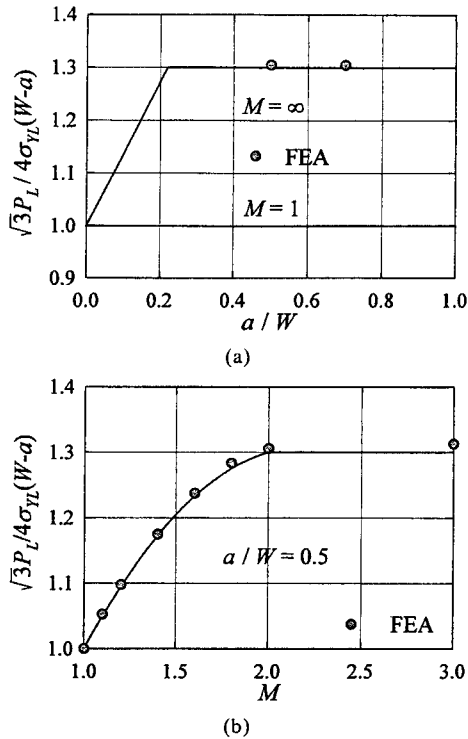
Fig. 8 Results for limit load P_L for the homogeneous and bimaterial M(T) specimens

specimens can be used regardless of the strength mismatch:

$$\eta_p^{VLL} = \eta_p^{CMOD} = m = 1 \qquad (23)$$

5. Concluding Remarks

In the previous two sections, J and CTOD estimation schemes for various fracture toughness testing geometries have been proposed by the simple slip line field considerations, for both homogeneous and strength-mismatched bi-material specimens. The following conclusions can be drawn from the present work.

(1) For toughness testing of homogeneous specimens, J and CTOD estimation schemes for non-standard testing specimens (such as the single-edge-cracked specimen in four-point bend, SE(PB), and middle crack tension specimen, M (T)) can be estimated based on the slip line field and FE limit analyses, as aforementioned.

(2) The J and CTOD estimation schemes for testing homogeneous specimens can be used for bi-

material specimens with interface cracks. It has been shown that the error induced by neglecting the mismatch effect is less than 10% for all cases considered. Such error is insignificant, considering many other factors involved in fracture toughness testing. This conclusion that the homogeneous testing procedures can be used to bi-material specimens with interface cracks provides significant advantages in estimating fracture toughness values for the heat affected zone of typical weldments or for the interface of bi-material specimens.

(3) While this paper gives tractable approximations to J and CTOD estimation equation only for deep cracks (for $0.45 \le a/W \le 0.7$), the results for shallow cracks in this paper may be still useful to investigate the constraint effect on toughness. Note that for strength mismatched bi-material specimens, the constraint effect results from two different sources: (i) geometry-induced, and (ii) mismatch induced. Such sources of constraint have been well described by authors elsewhere (Kim and Lee, 1996; Lee *et al.*, 1999; Lee and Kim, 1998). Thus the results presented in this paper, together with those authors' works, can provide sufficient backgrounds to investigate the effects of the geometry and the strength mismatch on fracture toughness for bi-material joints.

Although the strength mismatch does not affect the J and CTOD estimation schemes for bi-material specimens, it contributes different deformation capacities in two constituents. For instance, it is well known that the plastic deformation is more likely to concentrate on the lower strength material, when the crack locates between two materials having different strengths. Such concentration effect is clearly illustrated in Fig. 9 which shows the effect of the strength mismatch, M on J and CTOD portions of the lower strength material, J_L and δ_L. For bending geometries such as SENB specimens, a slight mismatch may be sufficient for the lower strength material to carry all deformations. It shows that when $M > 1.2$, the measured J or CTOD values result entirely from the lower strength material. On the other hand, for tension loading, such effect is rather gradual;
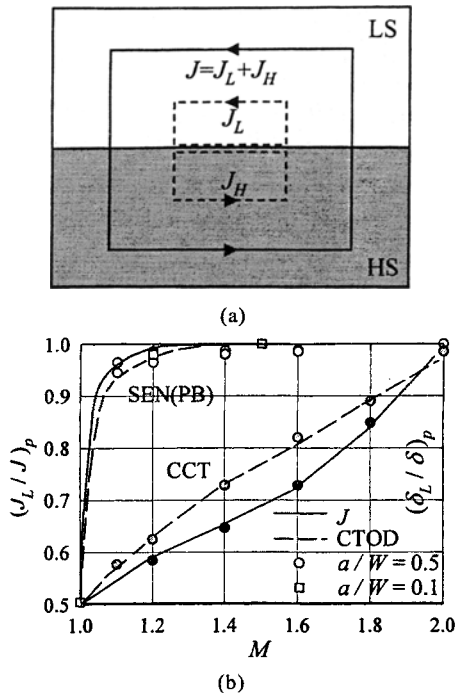
(a)

(b)

Fig. 9 Effect of the strength mismatch M on asymmetry of the J-integral and CTOD for bimaterial specimens: (a) path of J-integral (b) M vs. J_L/J for bimaterial M(T) and SE(PB) specimens

when $M > 1.2$, the measured J or CTOD values result entirely from the lower strength material. This trend has a significant implication of the J and CTOD testing as well as assessments of bimaterial joints. The measured (apparent) toughness values inferred from global quantities do not reflect actual (local) quantities, unlike homogeneous specimens. Thus for the bi-material specimens, the measured quantities must be carefully interpreted (possibly according to Fig. 9) to be used for structural assessments.

Acknowledgments

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